

P-20: Electronic Image Correction for Projection Displays

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Abstract

Optical distortions, resulting from lens characteristics, non-aligned projection and variations in the light source, decrease the quality of projection displays. Knowledge of the sources and characteristics of these distortions allows their electronic correction. The integration of electronic image correction in the display controller IC allows high quality projection without additional components.

1. Introduction

For image presentation with a projection system, the electronic image information is presented on a digital display device, e.g. an LCD or DMD. A lens system is used to project the image onto a screen. For image acquisition with an electronic camera a scene in the real world is projected by a lens system on an image sensor and transferred to an electronic signal.

In the projector as well as the camera an optical system and an electronic system work closely together. Consequently, during the design of the electronic system the characteristics of the optical system and vice versa have to be considered. Furthermore, one system can compensate the deficiencies of the other. Due to the advances in VLSI technology leading to increasing computing power for decreasing cost, it is the electronic system that can compensate more and more optical distortions.

Optical distortions have been accepted for some applications because only qualitative measurement but no geometric accuracy is required. For other applications, e.g. image projection, geometric accuracy is not absolutely necessary but becomes a quality feature. A further range of applications really requires geometric accuracy.

In this paper we consider optical distortions for image presentation and how image correction can be integrated in a display controller IC. The next chapter will present sources of optical distortion and thus define the requirements for electronic correction. In chapter three we present an architecture for electronic distortion correction and chapter four discusses an ASIC implementation currently under development.

2. Sources of Optical Distortion

Notation: Undistorted coordinates will be nominated as (x,y) and distorted coordinates as (x',y') . Origin is the optical axis.

2.1. Lens Distortion

Real lens systems are non-linear and introduce a radial shift to the position of the image pixel. This shift can result in a radial distortion called pincushion or barrel distortion. Figure 1 shows the effect a barrel distortion has on a rectangular grid of lines.

Radial distortion can be described as a coordinate transformation [1]. The image information from position (x, y) is transformed to position (x', y') according to:

$$x_b' = x + k_1 \bar{x} d^2 + k_2 \bar{x} d^4 + k_3 \bar{x} d^6 + P_1(2\bar{x}^2 + d^2) + 2P_2 \bar{x} \bar{y} \quad (1)$$

$$y_b' = y + k_1 \bar{y} d^2 + k_2 \bar{y} d^4 + k_3 \bar{y} d^6 + P_2(2\bar{y}^2 + d^2) + 2P_1 \bar{x} \bar{y} \quad (2)$$

with $\bar{x} = x - u_0$, $\bar{y} = y - v_0$, $d^2 = \bar{x}^2 + \bar{y}^2$ and (u_0, v_0) as the optical center of the image, also denoted as the optical axis.

The first order distortion, denoted as k_1 , accounts for the mayor part of the distortion. A simplified model of barrel distortion disregards the other coefficients and is written as

$$x_b' = x + k_1 \bar{x} d^2 \quad (3)$$

$$y_b' = y + k_1 \bar{y} d^2 \quad (4)$$

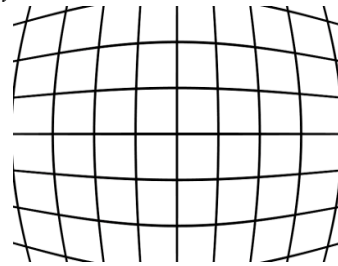


Figure 1: Barrel distortion of a rectangular grid of lines

2.2. Rotation

A rotation between the display device and the projection screen can be expressed by multiplying the original coordinates with the rotation matrix R

$$R = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \quad (5)$$

giving

$$x_r' = x \cos \alpha - y \sin \alpha \quad (6)$$

$$y_r' = x \sin \alpha + y \cos \alpha \quad (7)$$

2.3. Projection Systems

A projection system normally is designed to project the image of a display device on a vertical projection screen. As a projector is usually placed on a desk and projects slightly upwards, a certain projection angle is often considered, e.g. by shifting the display device from the center of the optical axis. This is illustrated by Figure 2.

Assuming undistorted projection, a picture element (x_d, y_d) of the display device is projected to a position on the screen (x_p, y_p) , which relate by a magnification factor m .

$$(x_p, y_p) = (m \cdot x_d, m \cdot y_d) \quad (8)$$

According to Figure 2, we define a coordinate system x_p, y_p, z . The projection screen \bar{P} is located in the origin and spans along the x - and y - coordinates.

$$\bar{P} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} \quad (9)$$

For typical projection applications, the dimensions of the lens are much smaller than the projection distance D . Then, the light source can be assumed to be a single point. In Figure 2, the projection lens is located at position $(0,0,-D)$ on the z -axis. A ray of light from the lens at position $(0,0,-D)$ towards the projected image pixel at $(x_p, y_p, 0)$ is defined as

$$\vec{r} = \begin{pmatrix} r \cdot x_p \\ r \cdot y_p \\ D(r-1) \end{pmatrix} \quad (10)$$

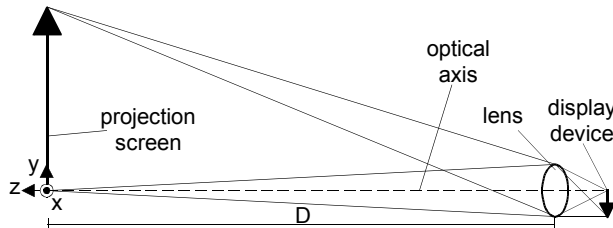


Figure 2: Projection system

2.4. One-Dimensional Keystone Distortion

When the angle of projection is different from the default angle, the optical system behaves as if it projects on a tilted screen (see Figure 3).

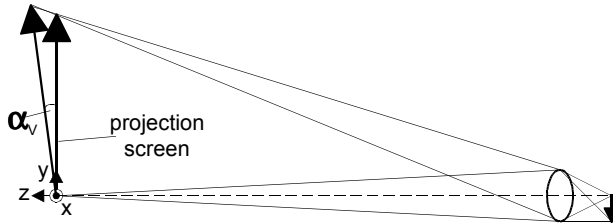


Figure 3: Tilted projection system

The rays towards the top of the screen have a larger distance from the lens so that the projected image becomes wider at the top. This so called keystone distortion results in a trapezoid-like image as depicted in Figure 4.

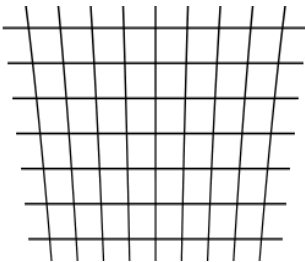


Figure 4: Keystone distortion of a rectangular grid of lines

A projection screen, tilted from the y -axis by an angle of α_v (compare Figure 3) is described by multiplying \vec{P} (equation 9) with the rotation matrix

$$R_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_v & -\sin \alpha_v \\ 0 & \sin \alpha_v & \cos \alpha_v \end{pmatrix} \quad (11)$$

giving

$$\vec{P}' = R_v \cdot \vec{P} = \begin{pmatrix} u \\ v \cos \alpha_v \\ v \sin \alpha_v \end{pmatrix} \quad (12)$$

The intersection of the projection rays \vec{r} towards the undistorted coordinates (x_p, y_p) with the tilted screen \vec{P}' has to be determined to calculate the distorted coordinates (x_k, y_k) .

Setting $\vec{P}' = \vec{r}$ and transforming the coordinates from the original coordinate system to the coordinate system of the tilted screen gives the keystone distorted coordinates as

$$p'_k = \begin{pmatrix} x'_k \\ y'_k \\ 0 \end{pmatrix} = R \cdot p' = \begin{pmatrix} \frac{x_p \cos \alpha_v}{\cos \alpha_v - \frac{y_p}{D} \sin \alpha_v} \\ y_p \\ \frac{y_p}{\cos \alpha_v - \frac{y_p}{D} \sin \alpha_v} \\ 0 \end{pmatrix} \quad (13)$$

2.5. Further Geometric Distortions

Further geometric distortions include two-dimensional keystone distortion, when the projection is performed upwards and from a side, and projection onto curved surfaces. They can be described with the same approach as above and the resulting distortion can be calculated.

2.6. Brightness Variation

Due to variations in the light source and the lens system the brightness of a projected image normally is non-uniform. The brightness of a projected image can be measured at 9 points according to Figure 5.

E1 ○	E2 ○	E3 ○
E4 ○	E5 ○	E6 ○
E7 ○	E8 ○	E9 ○

Figure 5: Brightness measurement positions

Brightness uniformity G can be defined as the ratio between the brightest and darkest of these nine measurement positions. Measurements showed that brightness uniformity can be lower than 60% [2].

While this uniformity only considers the nine measurement positions, the difference between the brightest and the darkest spot on the screen is usually even larger.

Optical distortion adds further brightness variation, as projection from an angle is less intensive as direct projection. The brightness distribution for distorted projection can be calculated as the intensity $I(x_p, y_p)$ for undistorted projection multiplied with the ratio between area in distorted $(\Delta x_k \cdot \Delta y_k)$ and undistorted $(\Delta x_p \cdot \Delta y_p)$ projection.

Thus, the relative brightness $i(x_p, y_p)$ is equal to the inverse of the derivative of the coordinate transformation between of the optical distortion.

$$i(x_p, y_p) = \frac{1}{\frac{d}{dx_p} x_k(x_p, y_p) \cdot \frac{d}{dy_p} y_k(x_p, y_p)} \quad (14)$$

For example, one-dimensional keystone correction according to equation (13) leads to a relative brightness distribution of

$$i_k(x_p, y_p) = \cos \alpha_v \left(1 - \frac{y_p}{D} \tan \alpha_v \right)^3 \quad (15)$$

3. System Architecture for Distortion Correction

Algorithms for distortion correction on general purpose computers have been discussed in literature [3]. Unless a very powerful computer is used, no real-time distortion correction is possible with this approach.

In this chapter, a VLSI architecture for correction of geometric and brightness distortions is described. Both corrections are performed independently from each other.

3.1. Correction of Geometric Distortions

For undistorted projection, the relation between a position (x_d, y_d) on the display device and the projected position is just a magnification (compare equation 1).

For distorted projection the image information that should be displayed at position (x_p, y_p) appears at position (x'_p, y'_d) . To compensate this distortion, the transformation $(x_p, y_p) \rightarrow (x'_p, y'_d)$ must be known and considered for the display device.

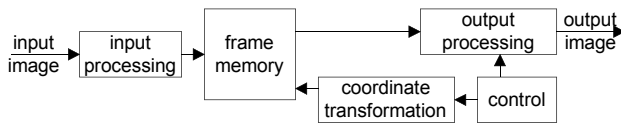


Figure 6: Architecture for geometric correction

A circuit architecture for this compensation is depicted in Figure 6. First, the input image is stored in a frame memory, to allow random access to the image information. Digital output devices are usually addressed line-by-line. For each output pixel, a coordinate transformation calculates, where this pixel will be displayed on the output screen. Then the image information corresponding to the screen position is fetched from the frame memory and displayed on the output device.

3.2. Coordinate Transformation

The coordinate transformation must be able to cover the different distortion patterns or even combinations of two or more distortions discussed in Chapter 2.

For a given distortion, the trigonometric terms appearing in the equations of Chapter 2 are constant. Therefore, equation 13 can be written as

$$p''_k = \begin{pmatrix} \frac{a_{01} x_p}{b_{00} + b_{10} y_p} \\ \frac{a_{10} y_p}{b_{00} + b_{10} y_p} \\ 0 \end{pmatrix} \quad (16)$$

with constant factors $a_{01}, a_{10}, b_{00}, \dots$ depending on the angle α_v .

A generalized equation for coordinate transformation is the rational function

$$f(x,y) = \frac{a_{00} + a_{01}x + a_{10}y + a_{02}x^2 + a_{11}xy + a_{20}y^2 + \dots}{b_{00} + b_{01}x + b_{10}y + b_{02}x^2 + b_{11}xy + b_{20}y^2 + \dots} \quad (17)$$

However, due to the division, this function leads to a rather complex hardware implementation. Furthermore equation 17 is undefined at the roots of the denominator and can become discontinuous. As the image distortions discussed previously usually do result in steady functions with steady derivatives, equation 17 can be approximated by a polynomial function

$$f(x,y) = c_{00} + c_{01}x + c_{10}y + c_{02}x^2 + c_{11}xy + c_{20}y^2 + \dots \quad (18)$$

This function can be calculated with lower hardware effort. In the proposed VLSI architecture, transformation of x- and y-coordinates are performed by independent function blocks.

3.3. Aliasing

In many cases, the coordinate transformation will not give the position of an existing input pixel but will result in a position between two pixel. The easiest approach for calculating an output pixel would be to fetch the nearest existing pixel. However, the results of this simple approach are not acceptable.

Bilinear filtering gives significantly better results, taking into account the neighbouring pixel and weighting them according to the sub-pixel position.

Optimal results can be achieved by anti-aliasing filters. This approach considers, that an image signal is a discrete-time signal, sampled with a certain (spatial) frequency. Scaling such a signal is equivalent to resampling it with another frequency [4].

3.4. Memory Organization

For the frame memory a trade-off between price and memory access has to be found. To avoid so-called frame-tears, i.e. artifacts due to overlapping write and read pointers, two frames must be stored. Storing two UXGA images with 1200 lines, 1600 pixel per line and 24 bit per pixel requires a memory capacity of 88 Mbit.

This capacity can be provided by a single DDR-DRAM [5]. However, this memory does not allow a fast random access. Thus, an on-chip cache for the image data is required.

The output of the geometry correction is calculated line-by-line. For geometry correction, as discussed in Chapter 2, the transformed pixel coordinates (denoted as 'x') follow a scan-line as shown in Figure 7(a). For most practical applications, the slope of this scanline is relatively flat, which can be used for the cache control. For calculation of an output pixel, the input data (denoted as 'o') in a window around the calculated output position are required (compare Figure 7(b)). The size of the window depends on the aliasing filtering. Typical values for n_H and n_V are between 2 (bilinear filtering) and 8 to 16.

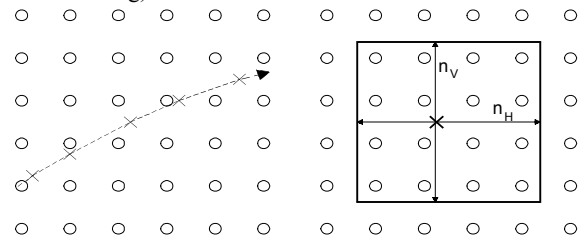


Figure 7: (a) Scanline of output coordinates (x) (b) Window of required input pixel (o)

The cache control uses the result of the coordinate transformation and fetches segments of data around the scanline [6]. Unlike in [6], the segments of the cache do not need to have the same vertical position but might have vertical displacement.

The cache segment size is determined by the organization of the external memory and the available bandwidth. As an example, we assume one external DDR-DRAM with a 32bit bus, 222 MHz bus frequency and a burst-length of 8. Then, one DRAM access contains $8 \cdot 32 = 256$ bit, corresponding to ten pixel with 24bit resolution. Thus, a segment width C_H of 10 pixel is sensible.

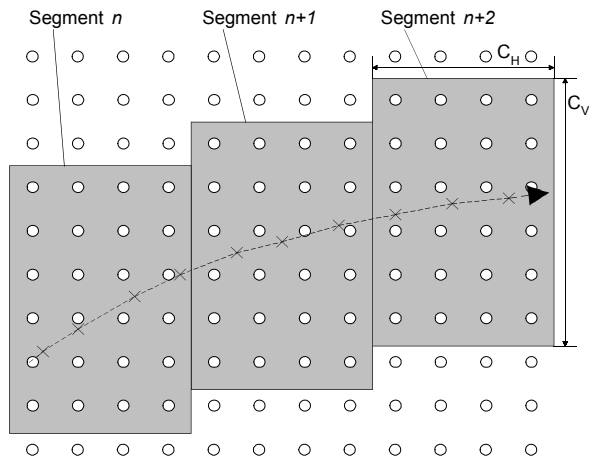


Figure 8: Cache segments around scanline

With a bandwidth of 14.2 Gbit/s ($32\text{bit} \cdot 222\text{MHz} \cdot 2$ words/clock) and assuming 25% overhead due to task switches, blanking and refresh, an XGA image with 60 Hz frame rate can be transferred ten times. One transfer is needed for writing, so the vertical segment size C_V can be nine.

With the size of the cache segments and the size of the anti-aliasing filter the maximum slope that can be corrected can be determined. Future DDR-DRAMs will have higher bandwidth, thus allowing larger image sizes to be processed and larger vertical segment sizes.

3.5. Brightness Correction

The correction of brightness variation is performed after geometry correction. For each output pixel, a translucency can be defined. The brightness of the output pixel is then reduced by the defined value.

Brightness variations due to the construction of a projection system can be calculated or measured and used for all devices of the production series. Variations due to tolerances or variations of the optical elements can be measured after production.

When geometry distortion is the cause of a brightness variation, the translucency map can be calculated as discussed above. Also, a projector specific translucency map can be modified according to the brightness variation by geometry distortion.

4. System Integration

A cost effective integration of electronic image correction into a display system is its combination with the display controller IC. A high-end display controller already contains a frame memory for frame-rate conversion and de-interlacing [7]. This frame memory can be shared for image correction. The integration of an embed-

ded CPU with its SRAM and an USB controller allows to further reduce system cost.

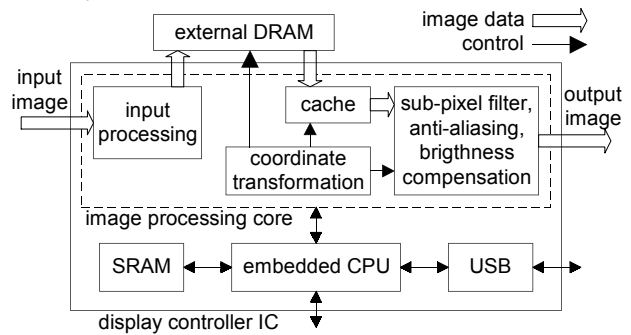


Figure 9: System architecture

The complete system architecture for compensation of the different optical distortions is depicted in Figure 9. An ASIC implementation of the presented architecture is currently under design.

An example for an image processed with the described architecture is depicted in Figure 10. It shows a combination of keystone and pincushion correction.



Figure 10: Processed image

5. References

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